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# Introduction

The National Council of Teachers of Mathematics (NCTM) Vision for School Mathematics invites us to “*Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction.*” It goes on to describe how this may take place in the classroom:

*Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it.* (National Council of Teachers of Mathematics (NCTM). *Principals and Standards for School Mathematics*. Reston, VA: NCTM, 2000).

Our goal in writing this book is to provide examples of how a symbolic geometry system, Geometry Expressions, can begin to make this happen. Geometry Expressions provides a playground where students can discover their own mathematics. They will begin to see mathematics as something that is created, not just a set of facts made up long ago. Once students take ownership of their mathematics, they will be more apt to “work productively and reflectively, with the skilled guidance of their teachers.”

The graphical, interactive nature of Geometry Expressions brings life into a field that might otherwise seem irrelevant. The symbolics embedded in Geometry Expressions offer an algebraic view of the mathematics in concert with a geometric view, blurring the artificial line between the two. The smooth interface between Geometry Expressions and Computer Algebra Systems (CAS) adds another powerful resource for solving problems. These technologies can work together to change the way mathematics is done, in the same way that technology has changed the way architectural design is done; with computers managing the details while humans create the grand vision.

The units presented in this book are a jumping-off point for using Geometry Expressions in the classroom. Use the units to gauge the potential of this powerful software, and as a guide to applying Geometry Expressions in your own classroom. We trust that you will enjoy using the units and the software.



Name: \_\_\_\_\_

Date: \_\_\_\_\_

## A Quick Review of Functions

We are about to look at functions in a new way: in a form called “Parametric Functions.”

But before we do that, we need to quickly review the old way!

Start by opening a new file with Geometry Expressions. Turn on the axis (look at the icon bar along the top) if it is not already on.

1. Create a parabola.

Click on the **Draw Function** icon 

Leave the type as *Cartesian*

Type  $x^2$  in the Y= box

You should see the familiar parabolic graph of  $y = x^2$

Now draw a point on the graph.

Click on the **Draw Point** icon 

Move your cursor over the parabola. It will change color when you are directly over it.

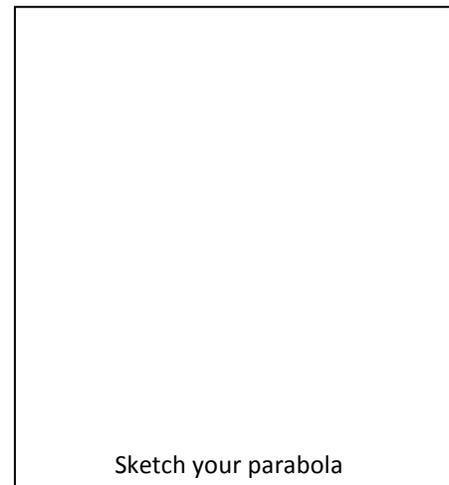
Select the parabola.

Click on the **Draw Select** icon  when you are finished drawing your point.

2. Who is in control?

The most important characteristic of a function is that one variable is in control – the control variable – while the other depends on the value of the control variable.

Click on the point and carefully drag your cursor to the left and right. What happens to the point?



Now carefully drag up and down. What happens now?

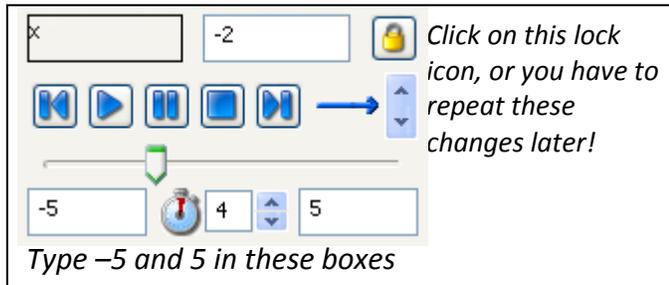
Which variable is in control,  $x$  (left and right) or  $y$  (up and down)?

Hold down the shift key and select the point, then select the parabola.

Click on **Constrain Point proportional along curve.** 

Change the variable from  $t$  to  $x$ .

$x$  is now listed in the Variable Tool Panel, though  $y$  is not. Select it, and change the animation control box to look like this:



Drag the slider bar, or click on the play button to see how  $x$  controls the position of the point.

Now, select the point.

Click on **Calculate Symbolic Coordinates.** 

What are the results?

What does the second coordinate depend on?

Which variable is dependent?

3. It is often said that a graph is a function if it passes the vertical line test. Does the parabola pass the vertical line test?

Click on **Draw Infinite Line**  and draw a line in the window.

Click on the **Select Arrow**  when you are finished drawing the line.

Select the line and Click on **Constrain Direction**. 

Look in the lower right corner of the Geometry Expressions window. It will tell you if you are in degrees or radians.

If you are in degrees, type 90.

If you are in radians, click on the  $\pi$  icon in the symbols window, then type  $/2$ .

Drag the line across the parabola. Does the line ever cross the parabola more than once?

Hold down shift, select the point, and select the line. Click on **Constrain Incident**. 

This will place the point on the vertical line.

Select variable  $x$  from the Variables Tool Panel. Click Play.

The definition of a function is this:

A function is a mapping between two sets such that each member of one set corresponds with only one member of the other set.

The vertical line test demonstrates that each value of  $x$  corresponds with only one value of  $y$ .

Does the parabola pass the vertical line test?

4. Let's look at some other functions.

First make sure you are in Radian mode.

Click Edit on the menu bar.

Click on Preferences.

Click on Math.

Change Angle Mode to Radians.

Look at the Geometry Expressions window and find the equation of your parabola:

$$Y = X^2$$

Double click on it.

Type  $\sin(x)$

What are the coordinates of the point now?

Select  $x$  in the variables window and press play.

Does  $y = \sin(x)$  pass the vertical line test?

Repeat with  $\sqrt{x}$ ,  $\ln(x)$ , and  $2^x$

(Hint: Make sure that your point is in quadrant I or IV (so that  $x > 0$ ) before changing the function equation. Otherwise, it may disappear).

	Coordinates of Point	Pass vertical line test?	Is it a function?
$y = x^2$			
$y = \sin(x)$			
$y = \sqrt{x}$			
$y = \ln(x)$			
$y = 2^x$			

$x^2$ ,  $\sin(x)$ ,  $\sqrt{x}$ ,  $\ln(x)$  and  $2^x$  are all **rules** for finding the  $y$  coordinate. Every function must include a rule that explains how the control variable is used to calculate the dependent variable.

5. Summary:

Functions in this lesson are sets of points with these properties:

$x$  is the \_\_\_\_\_ variable

$y$  is the \_\_\_\_\_ variable

A \_\_\_\_\_ explains how  $x$  and  $y$  are matched up.

Every value for the \_\_\_\_\_ variable corresponds with only one value of the \_\_\_\_\_ variable.

The graph of the function passes the \_\_\_\_\_ test.